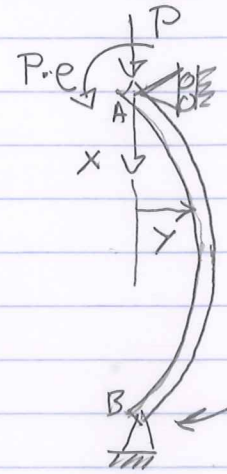
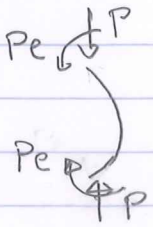
36. Columns that carry additional loads that cause bending

Eccentrically loaded columns



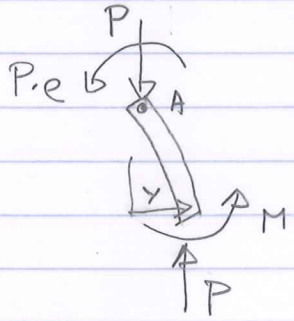
e is the eccentricity (distance from centroid where P is applied)

column length ℓ



Support reaction has same eccentricity.

Consider a FBD of an upper part of the deformed column. Because of the symmetry, no shear force exists.



$$\Sigma M_A = 0 = M + P \cdot y + P \cdot e = 0 \quad \Rightarrow M = -Py - Pe$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{Py}{EI} - \frac{Pe}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = -\frac{Pe}{EI}$$

$$y = a \cdot \sin\sqrt{\frac{P}{EI}}x + b \cdot \cos\sqrt{\frac{P}{EI}}x - e$$

$$y(x=0) = 0 \Rightarrow b = e$$

$$y(x=\ell) = 0 \Rightarrow a = \frac{e\left(1-\cos\sqrt{\frac{P}{EI}}\ell\right)}{\sin\sqrt{\frac{P}{EI}}\ell} = e \cdot \tan\sqrt{\frac{P}{EI}}\frac{\ell}{2}$$

$$y = e\left(\tan\sqrt{\frac{P}{EI}}\frac{\ell}{2} \cdot \sin\sqrt{\frac{P}{EI}}x + \cos\sqrt{\frac{P}{EI}}x - 1\right)$$

Because of the symmetry, y_{max} occurs at $x = \frac{\ell}{2}$. After some algebra,

$$y_{max} = e\left(\sec\sqrt{\frac{P}{EI}}\frac{\ell}{2} - 1\right)$$

The EI in this equation is replaced by the EI from $P_{cr} = \pi^2 EI/\ell^2$, the buckling load for the pinned-pinned column, to give

$$y_{max} = e\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} - 1\right)$$

from which it is evident that $y_{max} \to \infty$ as $P \to P_{cr}$.

The maximum bending couple also occurs at mid-span and is given by

$$M_{max} = P(e + y_{max})$$

which also $\to \infty$ as $P \to P_{cr}$.

The Pe part of M_{max} is due to the Pe couple at each end of the column (which produces a constant moment of Pe along the column). The $P \cdot y_{max}$ part of M_{max} is the additional moment due to P acting through the lateral deflection of the column.

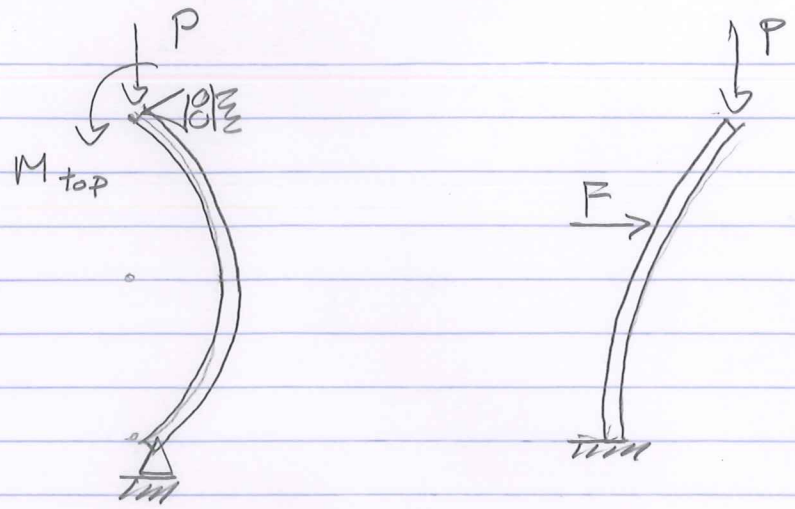
The maximum bending couple can also be written as

$$M_{max} = Pe \cdot \sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}$$

where the sec term can be interpreted as a moment amplification factor. For example, if $P = P_{cr}/2$, $M_{max} = Pe \cdot 2.25$, a significant amplification.

Other cases

Consider other cases in which loads independent of P act on the column.



Again, the bending couple in the column consists of a primary part due directly from M_{top} (left case) or F (right case) and a secondary part P.y. The primary part is amplified by the presence of P. If one is computing the maximum moment in the column for design purposes, it is important to consider the moment amplification, which can be very large as P approaches P_{cr}.